Squeeze Film Problems of Long Partial Journal Bearings for Non-Newtonian Couple Stress Fluids with Pressure-Dependent Viscosity

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According to the experimental work of C. Barus in Am. J. Sci. **45**, 87 (1893) [1], the dependency of liquid viscosity on pressure is exponential. Therefore, we extend the study of squeeze film problems of long partial journal bearings for Stokes non-Newtonian couple stress fluids by considering the pressure-dependent viscosity in the present paper. Through a small perturbation technique, we derive a first-order closed-form solution for the film pressure, the load capacity, and the response time of partial-bearing squeeze films. It is also found that the non-Newtonian couple-stress partial bearings with pressure-dependent viscosity provide better squeeze-film characteristics than those of the bearing with constant-viscosity situation.

Key words: Partial Bearings; Pressure-Dependent Viscosity; Non-Newtonian Fluids; Squeeze Films.

1. Introduction

Squeeze film effects arise when two lubricated surfaces approach mutually with a normal velocity. This phenomenon is important in many areas of bio-lubrication and engineering sciences, such as the animal joint, the reciprocating engine, machine tools, braking apparatus, etc. In the literature, the squeeze-film journal bearings lubricated with a Newtonian lubricant have been investigated by Pinkus and Sternlicht [2], Prakash and Vij [3], and Murti [4]. With the improvement of machine systems, the wide application of non-Newtonian lubricants has received great attention. General non-Newtonian lubricants can be observed such as the synovial fluids, polymer-thickened oils, and base lubricants blended with additives. For a better description of the flow behaviour of these kinds of non-Newtonian fluids, a non-Newtonian micro-continuum theory including the effects of couple stresses and body couples was developed by Stokes [5]. This micro-continuum theory is important for pumping fluids, such as bio-fluids, liquid crystals, and complex fluids. According to the thin-film lubrication theory by Pinkus and Sternlicht [2] together with the negligible body couples, the momentum equations, and the continuity equation of a Stokes non-Newtonian incompressible couple stress fluid by Stokes [5] for two-dimensional rectangular coordinates (x,z) can be written as

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}\left(\mu \cdot \frac{\partial u}{\partial z}\right) - \eta \cdot \frac{\partial^4 u}{\partial z^4}, \tag{1}$$

$$\frac{\partial p}{\partial z} = 0, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0. \tag{3}$$

In the above equations, p is the fluid pressure, u and w are the velocity components in the x- and z-directions, respectively, μ denotes the viscosity coefficient; in addition, η represents a new material constant responsible for the non-Newtonian properties of couple stress fluids. By applying this couple stress fluid model and assuming a constant viscosity (CV) of the fluid, many researchers have investigated the peristaltic transport problems and the squeeze film problems, such as the mechanisms of peristalsis by Srivastava [6] and Shehawey and Mekheimer [7]; the squeeze film plates with reference to human joints by Tandon and Jaggi [8] and Ahmad and Singh [9]; the squeeze film behaviour between parallel circular disks by Lin and Hung [10]; the squeeze film characteristics between a cylinder and a plate by Lin and Hung [11]; the squeeze film performances between a sphere and a plate by Naduvinamani et al. [12] and Elsharkawy and Al-Fadhalah [13]; and the squeeze film phenomenon in long partial journal bearing by Lin [14] and Lin et al. [15].

As reviewed in the above passage, the studies [6–15] have assumed that the fluid viscosity μ is constant, the peristaltic transport problems and the squeeze film problems are then analyzed. However, the viscosity of the fluid could depend upon the film pressure. From the contributions by Barus [1], a relationship for the exponential dependency of viscosity on the fluid pressure is provided through the equation

$$\mu = \mu_0 \exp(mp), \tag{4}$$

where μ_0 is the viscosity at ambient pressure and constant temperature, and m denotes the pressure-viscosity coefficient. By applying this exponential dependency of viscosity on the pressure, a Newtonian spherical squeeze film problem has been investigated by Gould [16]. In addition, by applying both the Stokes micro-continuum theory [5] and the relationship for the pressure-dependent viscosity (PDV) by Barus [1], the squeeze film mechanism between a sphere and a flat plate has been investigated by Lu and Lin [17]. However, the study for squeeze-film partial bearings is absent. Since the squeeze film mechanism of partial bearings is also important in engineering application [2, 14, 15], an advanced study is motivated.

In the present paper, we extend the study of squeeze film problems of long partial journal bearings for non-Newtonian couple stress fluids [14] with variable viscosity through the application of the micro-continuum theory by Stokes [5] together with the relationship for the exponential PDV by Barus [1]. By employing a small perturbation technique, a first-order closed-form solution for the film pressure, the load capacity and the response time of partial-bearing squeeze films has been derived. Comparing with those of the non-Newtonian bearing with CV situation, the effects of exponential PDV on the squeeze-film partial-journal bearings are presented and discussed through the variation of the pressure-dependent viscosity parameter and the non-Newtonian couple-stress parameter.

2. Analysis

Figure 1 presents the squeeze film configuration of a long partial journal bearing lubricated with a non-Newtonian couple stress fluid considering pressure-dependent viscosity, in which C denotes the radial clearance, and e is the eccentricity. The journal with radius R is approaching the partial bearing with a squeezing velocity: $-\partial h/\partial t$. It is assumed that the thin-film lubrication theory of hydrodynamic lubrication by Pinkus and Sternlicht [2] is applicable, the body couples for the couple stress fluids by Stokes [5] are negligible, and the dependency of viscosity on pressure is exponential [1]. Based upon these assumptions, the momentum equations, the continuity equation, and the variation of the viscosity with fluid pressure reduce to (1), (2), (3), and (4). According to the derivation in the squeeze-film problem between a sphere and a flat plate, the cylindrical-form of the non-Newtonian couple-stress squeeze-film Reynolds-type equation with PDV has been obtained by Lu and Lin [17]. Expressed in the rectangular-coordinate form, we have the following equation for the present study:

$$\frac{\partial}{\partial x}\left\{ f(h,l,m,p)\cdot\frac{\partial p}{\partial x}\right\} = 12\mu_0\frac{\partial h}{\partial t}, \tag{5}$$

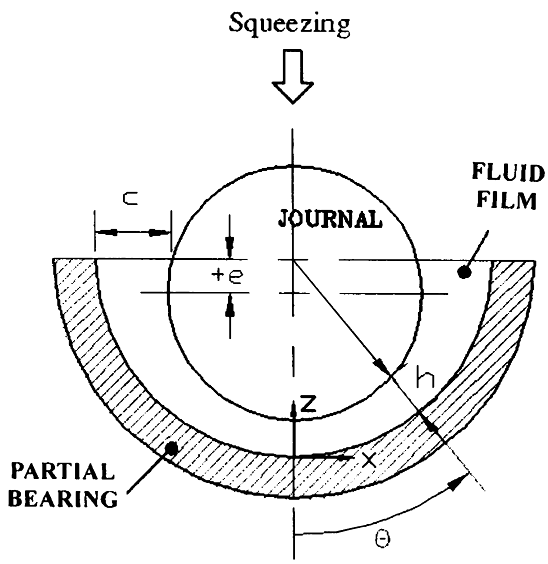

Fig. 1. Squeeze film configuration of a partial journal bearing lubricated with a non-Newtonian couple stress fluid considering pressure-dependent viscosity.

where the quantity l, the film thickness h, and the function $f = f(h,l,m,p)$ are given by:

$$l = (\eta/\mu_0)^{1/2}, \tag{6}$$

$$h = C - e\cos(\theta), \tag{7}$$

$$f = h^3 \mathrm{e}^{-mp} - 12l^2 h\mathrm{e}^{-2mp} + 24l^3 \mathrm{e}^{-2.5mp} \tanh\left(h\mathrm{e}^{0.5mp}/2l\right). \tag{8}$$

Using non-dimensional parameters and variables, the non-Newtonian squeeze-film Reynolds-type equation with PDV for the long partial journal bearing together with the boundary conditions can be written as:

$$\frac{\partial}{\partial\theta}\left\{f^*(h^*,N,G,p^*)\cdot\frac{\partial p^*}{\partial\theta}\right\} = -12\cos\theta, \tag{9}$$

$$\mathrm{d}p^*/\mathrm{d}\theta = 0 \text{ at } \theta = 0, \tag{10}$$

$$p^* = 0 \text{ at } \theta = \pm\pi/2, \tag{11}$$

where $p^* = pC^2/\mu_0 R^2(\mathrm{d}\varepsilon/\mathrm{d}t)$ is the non-dimensional squeeze film pressure, and $\theta = x/R$ represents the non-dimensional circumferential coordinate. Furthermore, the non-dimensional film thickness $h^* = h/C$ and the non-dimensional function $f^* = f^*(h^*,N,G,p) = f/C^3$ are defined by:

$$h^* = 1 - \varepsilon\cos(\theta) \tag{12}$$

$$f^* = h^{*3}\mathrm{e}^{-Gp^*} - 12N^2 h^* \mathrm{e}^{-2Gp^*} + 24N^3 \mathrm{e}^{-2.5Gp^*} \tanh\left(0.5h^*\mathrm{e}^{0.5Gp^*}/N\right). \tag{13}$$

In these equations, ε denotes the eccentricity ratio. In addition, the couple stress parameter N characterizes the effects of non-Newtonian rheology and the PDV parameter G identifies the effects of exponential pressure-dependent viscosity:

$$N = l/C, \tag{14}$$

$$G = m\mu_0 R^2(\mathrm{d}\varepsilon/\mathrm{d}t)/C^2. \tag{15}$$

It is seen that the non-Newtonian PDV Reynolds-type equation is a highly nonlinear differential equation. In order to simplify the problem and to obtain an approximately analytical solution for small values of the PDV parameter $0 \le G \ll 1$, a small perturbation technique for the film pressure following the similar procedure by Lu and Lin [17] is applied:

$$p^* = p_0^* + G\cdot p_1^*. \tag{16}$$

Substituting this equation into the non-Newtonian PDV Reynolds-type equation and neglecting the higher-order terms of the PDV parameter, we can obtain the equation governing the zero-order pressure p_0^* and the equation governing the first-order pressure p_1^*:

$$\frac{\partial}{\partial\theta}\left\{f_0^*(h^*,N)\cdot\frac{\mathrm{d}p_0^*}{\mathrm{d}\theta}\right\} = -12\cos\theta, \tag{17}$$

$$\frac{\partial}{\partial\theta}\left\{f_0^*(h^*,N)\cdot\frac{\mathrm{d}p_1^*}{\mathrm{d}\theta} + f_1^*(h^*,N)\cdot p_0^*\cdot\frac{\mathrm{d}p_0^*}{\mathrm{d}\theta}\right\} = 0, \tag{18}$$

where the non-dimensional functions $f_0^* = f_0^*(h^*,N)$ and $f_1^* = f_1^*(h^*,N)$ are expressed by

$$f_0^* = h^{*3} - 12N^2 h^* + 24N^3\tanh(h^*/2N), \tag{19}$$

$$f_1^* = -h^{*3} + 6N^2 h^*\left[4 + \mathrm{sech}^2(h^*/2N)\right] - 60N^3\tanh(h^*/2N). \tag{20}$$

It is noted that the differential equation governing the zero-order pressure p_0^* is the form of the non-Newtonian partial-bearing squeeze-film Reynolds-type equation with CV by Lin [14]. Now integrating the two differential equations, we can obtain approximate analytical solutions for the zero-order pressure and the first-order pressure:

$$p_0^* = 12\cdot g_0^*(h^*,N), \tag{21}$$

$$p_1^* = -144\cdot g_1^*(h^*,N), \tag{22}$$

where

$$g_0^*(h^*,N) = \int_\theta^{\theta=+\pi/2} \frac{\sin\theta}{f_0^*(h^*,N)}\mathrm{d}\theta, \tag{23}$$

$$g_1^*(h^*,N) = \int_\theta^{\theta=+\pi/2} \frac{\sin\theta\cdot f_1^*(h^*,N)}{f_0^{*2}(h^*,N)} \cdot g_0^*(h^*,N)\mathrm{d}\theta. \tag{24}$$

Integrating the squeeze film pressure acting upon the journal surface, we can obtain the load-carrying capacity:

$$W = \int_{\theta=-\pi/2}^{+\pi/2} p\cos\theta\cdot LR\mathrm{d}\theta, \tag{25}$$

where L denotes the length of the bearing. After performing the integration, the non-dimensional load-carrying capacity $W^* = WC^2/\mu_0 R^3 L(\mathrm{d}\varepsilon/\mathrm{d}t)$ can be expressed as

$$W^* = 12\cdot\int_{\theta=-\pi/2}^{+\pi/2} g_2(h^*,N,G)\mathrm{d}\theta, \tag{26}$$

where

$$g_2(h^*,N,G) = \frac{\sin^2\theta \cdot [f_0^*(h^*,N) - 12\cdot G\cdot f_1^*(h^*,N)\cdot g_0^*(h^*,N)]}{f_0^{*2}(h^*,N)}. \tag{27}$$

Now, we introduce the non-dimensional response time as

$$t^* = \frac{WC^2}{\mu_0 R^3 L}\cdot t. \tag{28}$$

Thereafter, one can derive the non-dimensional differential equation for the eccentricity ratio varying with the response time as

$$\frac{d\varepsilon}{dt^*} = \frac{1}{12\cdot\int_{\theta=-\pi/2}^{+\pi/2} g_2(h^*,N,G)\cdot d\theta}. \tag{29}$$

Integrating this differential equation together with the initial condition $\varepsilon(t^*=0)=0$, the response time t^* can be obtained:

$$t^* = 12\cdot\int_{\varepsilon=0}^{\varepsilon}\left[\int_{\theta=-\pi/2}^{+\pi/2} g_2(h^*,N,G)d\theta\right]\cdot d\varepsilon. \tag{30}$$

By applying the Gaussian-quadrature method of numerical integration by Faires and Burden [18], one can evaluate the values of the pressure, the load-carrying capacity, and the response time.

3. Results and Discussion

In the literature, there are several expressions proposed for the isothermal pressure-dependent viscosity of liquids. Many of the correlative methods are rather complicated such that the analysis and computation are not easy. On the other hand, the relationship of viscosity to pressure proposed by Barus [1] is commonly used and is relatively easy to investigate the problem. Therefore, the Barus equation is applied in the present study. However, this Barus equation is assumed under

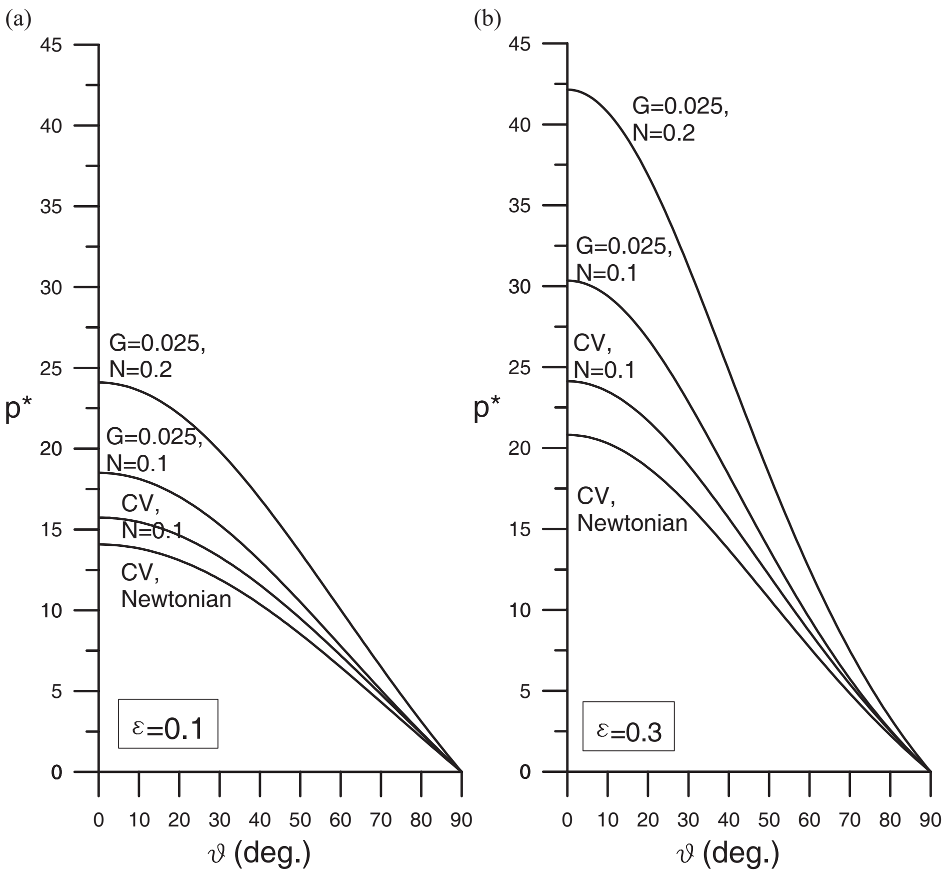

Fig. 2. Pressure distribution as a function of circumferential coordinate θ for different N and G; (a) $\varepsilon = 0.1$, (b) $\varepsilon = 0.3$.

the isothermal condition and is applicable at moderate pressure. According an excellent experimental contribution by Jones et al. [19], the pressure-viscosity coefficient of a number of liquid lubricants containing additives has been measured at 38 °C, 99 °C, and 149 °C, and gauge pressures up to $5.5 \cdot 10^8\ \mathrm{N/m^2}$. Following the similar procedure, the pressure-viscosity coefficient of couple stress fluids can be obtained for applications, such as in the area of matching gears, rolling bearings, and braking plates.

Based upon the above analysis, we have obtained a first-order analytical solution of long partial-bearing squeeze film problems for Stokes non-Newtonian couple stress fluids by considering the pressure-dependent viscosity. Two special cases can be obtained through the specific values of both the PDV parameter G and the non-Newtonian couple stress parameter N.

Case 1 for $G = 0$ and $N = 0$: it is the partial-bearing squeeze film problem by using Newtonian lubricants with CV by Pinkus and Sternlicht [2].

Case 2 for $G = 0$ and $N \neq 0$: it is the partial-bearing squeeze film problem by using Stokes non-Newtonian couple stress lubricant with CV by Lin [14].

To show both the effects of the pressure-dependent viscosity and the non-Newtonian couple stress cases, the following results are presented with the values of $G \neq 0$ and $N \neq 0$.

Figure 2 shows the film pressure distribution p^* as a function of the circumferential coordinate θ for different values of N and G. For the bearing with CV operating at the eccentricity ratio $\varepsilon = 0.1$, the effects of non-Newtonian couple stresses ($N = 0.1$) provide a higher pressure distribution as compared to the Newtonian-lubricant situation. For the bearing considering the pressure-dependent viscosity and applying the non-Newtonian couple stress lubricant ($G = 0.025$, $N = 0.1$; $G = 0.025$, $N = 0.2$), further higher distributions of the pressure are obtained. In addition, when the bearing operating at a larger eccentricity ratio $\varepsilon = 0.3$, the effects of PDV and non-Newtonian rheology on

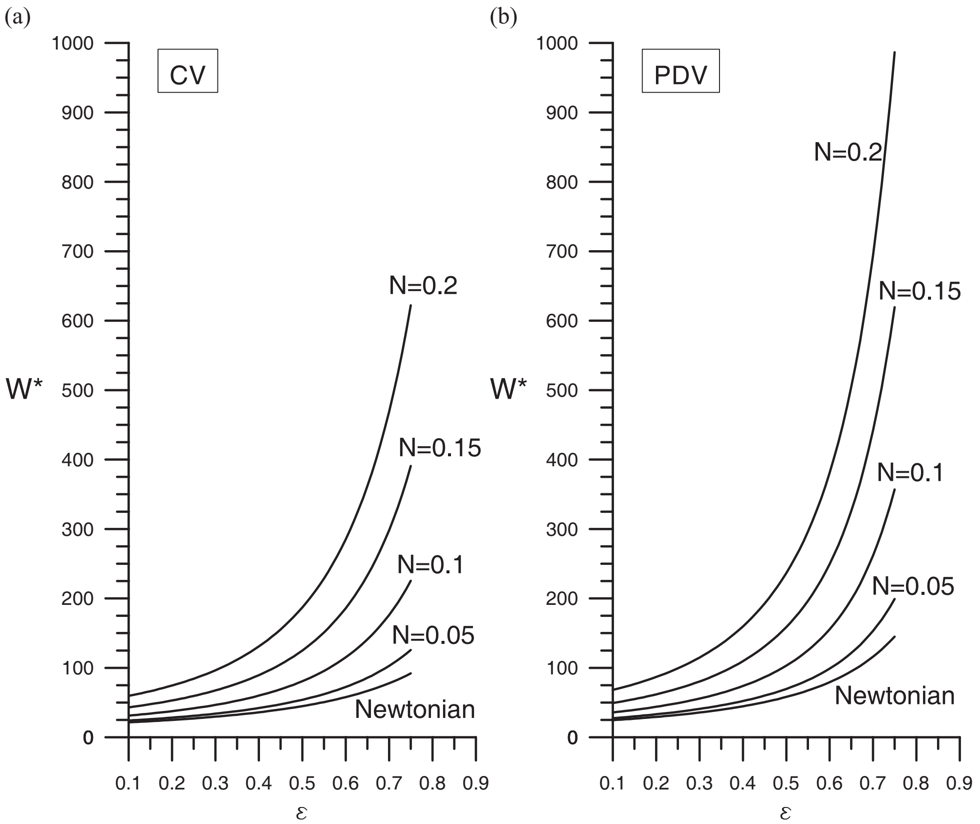

Fig. 3. Load capacity W^* as a function of eccentricity ratio ε for different N; (a) Constant viscosity, (b) $G = 0.025$.

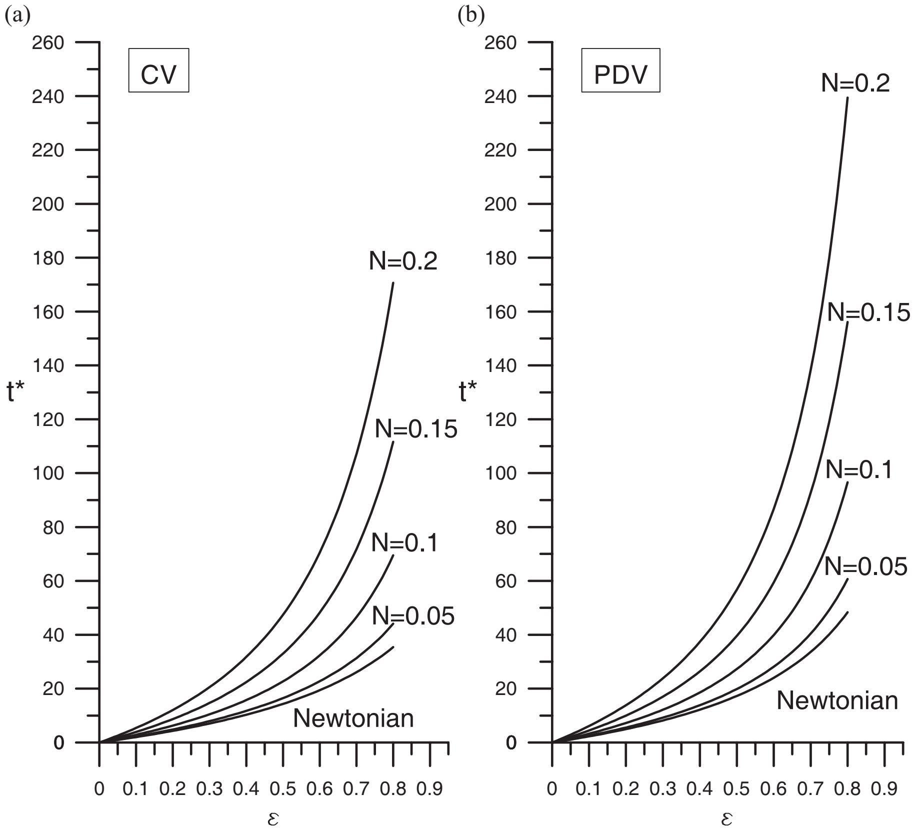

Fig. 4. Response time t^* as a function of eccentricity ratio ε for different N; (a) Constant viscosity, (b) $G = 0.025$.

the film pressure are more emphasized. Figure 3 displays the load-carrying capacity W^* as a function of the eccentricity ratio ε for different values of N. Under the CV case, the bearing with non-Newtonian couple stress fluids ($N = 0.05$, 0.1, 0.15, 0.2) provides higher loads as compared to the Newtonian-lubricant situation. When the lubricant with the PDV ($G = 0.025$) are considered, the effects of pressure-dependent viscosity provide further increments in the load capacity, especially for larger values of the non-Newtonian couple stress parameter N. Since the effects of PDV and non-Newtonian rheology result in a higher pressure distribution, the integrated load-carrying capacity is similar affected for the bearing lubricated with couple stress fluids and considering the pressure-dependent viscosity. Figure 4 presents the response time t^* as a function of the eccentricity ratio ε for different values of N. Under the CV case, the effects of non-Newtonian rheology ($N = 0.05$) are observed to provide a longer response time as compared to the Newtonian-lubricant situation. Increasing the non-Newtonian couple stress parameter ($N = 0.1$, 0.15, 0.2) lengthens more the values of the response time. When the effects of PDV ($G = 0.025$) are also considered, further values of the response time are lengthened for the partial-bearing squeeze films with non-Newtonian couple stress lubricants ($N = 0.05$, 0.1, 0.15, 0.2). Generally speaking, the non-Newtonian partial journal bearing considering the pressure-dependent viscosity results in better squeeze-film performances and provides longer bearing life.

4. Conclusion

By considering the pressure-dependent viscosity, the squeeze film problems of long partial journal bearings with non-Newtonian couple stress fluids has been extended in the present paper. Based upon the above analysis, results, and discussions, we can draw the conclusion as follows.

We have derived a closed-form solution of long partial journal non-Newtonian squeeze-film bearings with pressure-dependent viscosity for small values of the PDV parameter. Comparing with those of the non-Newtonian bearing with CV situation, the effects of exponential pressure-dependent viscosity provide better performance characteristics and prolong the squeezing life of partial bearings.

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